Structure formation and the origin of dark energy

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Cosmological constant *a.k.a.* dark energy problem is considered to be one major challenge in modern cosmology. Here we present a model where large scale structure formation causes spatially-flat FRW universe to fragment into numerous 'FRW islands' surrounded by vacuum. We show that this mechanism can explain the origin of dark energy as well as the late time cosmic acceleration. This explanation of dark energy does *not* require any exotic matter source nor an extremely *fine-tuned* cosmological constant. This explanation is given within classical general relativity and relies on the fact that our universe has been undergoing structure formation since its recent past.

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Several recent experimental observations [1] seem to strongly suggest that we live in the universe whose energy budget is dominated by contribution from a mysterious source which is otherwise invisible or missing in direct observations. This energy component is generally referred as *dark energy*. Furthermore, the dark energy component appears to have *negative* pressure. Result from supernova observations [2] that universe is undergoing a recent *acceleration* seems to confirm such peculiar behavior of dark energy.

Many attempts have been made in literature to understand the origin of dark energy (see [3] for some reviews). Arguably the most economical one is to introduce a non-zero *cosmological constant* in Einstein equation. However, there are several conceptual difficulties associated with it. Firstly, experimentally required value of cosmological constant turns out to be $\sim 10^{-123}$ in Planck unit. Such an extremely *fine-tuned* value of cosmological constant appears to defy any hope of possible explanation from a fundamental theory. Second conceptual problem is the so-called *cosmic coincidence* problem: why does energy density due to cosmological constant which remains unchanged, become comparable to changing matter energy density only at *current* epoch?

There have been other attempts where it is anticipated that late time acceleration could be due to a dynamical scalar field that evolves in a suitably engineered potential [4], due to super-horizon perturbations [5], from averaging of LTB models [6] or due to *back-reaction* effects of inhomogeneous structures in the universe [7, 8, 9]. In particular, using Buchert equations one can get acceleration by incorporating inhomogeneous back-reaction. However, same back-reaction term contributes *negatively* [8] to energy density thus fails to explain missing energy unless one also introduces strong *negative* spatial curvature. In brief, none of the current attempts can explain the origin of dark energy satisfactorily.

According to standard model of cosmology, continued expansion of the universe causes radiation to eventually decouple from matter. Subsequently, largely homogeneous matter distribution with small inhomogeneity starts collapsing to give rise current large scale structures. Initial phase of structure formation can be described by linear perturbation theory around homogeneous background. However, in later phase when structure formation enters non-linear regime such descriptions are insufficient. In this era, matter distribution consists of numerous dense regions surrounded by relatively rarer regions. For simplicity here we make *sharp-boundary* approximation for the matter distribution during later period of structure formation such that matter is contained within spherical, homogeneous regions surrounded by empty space. Boundary of such spherical regions, however, needs to be continuously refined as on-going structure formation continues to cause dense regions to become denser. In other words, in this model structure formation along with sharp-boundary approximation causes FRW universe to fragment into numerous 'FRW islands' with shrinking boundary that are surrounded by vacuum (see Fig.1). We will show that this mechanism can explain the origin of dark energy as well as the late time cosmic acceleration.

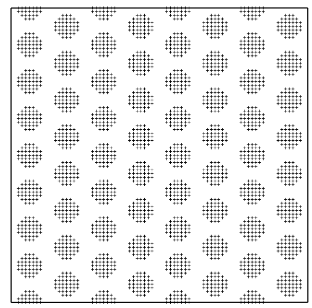

FIG. 1: A simple illustration of late time universe where structure formation causes FRW universe to fragment into numerous 'FRW islands'. Each FRW island (shaded region) is surrounded by vacuum metric.

In the standard model of cosmology, spacetime describing our universe is assumed to be foliated by homogeneous and isotropic spatial hyper-surfaces parametrized by a global time. We imagine such an observer who treats

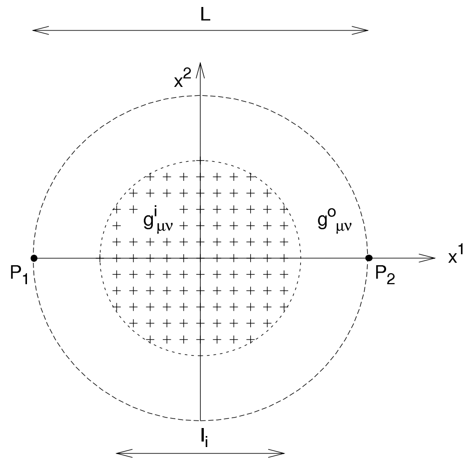

FIG. 2: A spherical patch of universe of coordinate diameter L which was homogeneous at the beginning of structure formation. In a later period the matter distribution, by means of sharp-boundary approximation, is contained within a spherical region (shaded) of coordinate diameter l_i.

the spatial hyper-surfaces as homogeneous and isotropic during *entire* evolution of universe and accordingly measures distance using an average spatially-flat Friedman-Robertson-Walker (FRW) metric

$$ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2 \ , \tag{1}$$

where $a(t)$ is the *scale factor*. We refer this observer as observer A. We consider another observer, say observer B, who uses same time parametrization of the spatial hyper-surfaces as observer A, but is careful to consider the effects of structure formation. In particular, to measure distance during structure formation observer B uses flat FRW metric $g^i_{\mu\nu} = diag(-1, a_i^2, a_i^2, a_i^2)$ inside the spherical regions containing homogeneous matter distribution otherwise uses vacuum metric $g^o_{\mu\nu}$. To illustrate this, let's consider a spherical patch of universe of coordinate (also co-moving for observer A) diameter L which is homogeneous at the beginning of structure formation (see Fig.2). The patch then begins to undergo structure formation such that at a given time collapsing matter distribution, by means of sharp-boundary approximation, is contained within a spherical region of coordinate diameter $l_i(t)$ with $0 < l_i \leq L$. We *assume* that the vacuum metric $g^o_{\mu\nu}$ describing the spherically symmetric empty space created due to structure formation, is *static*. (This would be the case if we ignore the presence of neighbouring patches, as then *Birkhoff's theorem* would imply spherically symmetric vacuum metric is static.) The vacuum metric $g^o_{\mu\nu}$ however is inhomogeneous as can be seen from *continuity* of metric at the boundary of inside region.

To have a precise notion we define *average* FRW metric (1) such that the *proper* distance between the points P_1 and P_2 (see Fig.2) as measured by observer A is *equal* to the proper distance measured by observer B. Without loss of generality we assume that the points lie on x^1-axis. Given elementary proper distance in a spatial hyper-surface is $|\sqrt{g_{jk}dx^j dx^k}|$ where j, k represent spatial coordinates, the definition leads to

$$aL = a_i l_i + 2\int_{l_i/2}^{L/2} \sqrt{g^o_{11}(x^1, x^2, x^3)}dx^1 \ . \tag{2}$$

Using the definition, we can also compute relation between expansion rates of the points as measured by both observers. In particular,

$$\begin{aligned} \dot{a}L \ &= \ \frac{d}{dt}\int_{-L/2}^{L/2} \sqrt{g_{11}(x^1, x^2, x^3)}dx^1 \\ &= \ \dot{a}_i l_i + a_i \dot{l}_i - \dot{l}_i\sqrt{g^o_{11}(l_i/2, 0, 0)} = \dot{a}_i l_i \ , \end{aligned} \tag{3}$$

where over-dot denotes derivative *w.r.t.* to time t. In last line, we have used continuity of metric solution *i.e.* $g^o_{11}(l_i/2, 0, 0) = g^i_{11} = a_i^2$. We may note that to have an expanding average metric, the metric of the inside region must also be expanding. However, proper volume of the inside region can still decrease as its coordinate diameter shrinks. We now define average energy density ρ for observer A, by requiring that at any given time total energy contained within the patch of coordinate diameter L is *same* as measured by observer B. In particular, if observer B measures energy density of inside region to be ρ_i and of outside region to be zero then

$$\rho = \left(\frac{a_i l_i}{aL}\right)^3 \rho_i =: n^3 \rho_i \ . \tag{4}$$

As defined, n^3 is the *fraction* of total proper volume occupied by matter distribution. Physically, parameter n is a measure of *amount* of structure formation and satisfies $0 < n \leq 1$. In particular, $n = 1$ implies there are no underlying structures. This can be seen from the equation (2). As we will see, equations (3) and (4) form the backbone of arguments presented here.

Let's imagine that both observers want to confront their respective Einstein equations with experimental data. Observer A performs separate observations to measure expansion rate as well as energy density. However, it turns out that to make a right balance, observer A needs to postulate an extra *invisible* component in Friedmann equation for the average metric *i.e.*

$$3\left(\frac{\dot{a}}{a}\right)^2 = 8\pi G(\rho + \rho_{DE}) \ , \tag{5}$$

where G is Newton's constant and ρ_{DE} denotes *dark energy* component. On the other hand observer B uses standard Friedmann equation for the region containing homogeneous matter distribution and uses vacuum Einstein equation for the remaining region. In particular,

Friedmann equation for observer B is

$$3\left(\frac{\dot{a}_i}{a_i}\right)^2 = 8\pi G\rho_i \ . \tag{6}$$

Equipped with the details of underlying structures *i.e.* using equations (3), (4) and Friedmann equation (6), observer B can derive Friedmann equation for average FRW metric (1), given by

$$3\left(\frac{\dot{a}}{a}\right)^2 = 8\pi G\left[\rho + \left(\frac{1}{n} - 1\right)\rho\right] \ . \tag{7}$$

One may note that right hand side of equation (7) has an extra energy density component apart from average energy density ρ. Thus, comparing equations (5) and (7), observer B can *derive* the expression of dark energy that observer A should perceive

$$\rho_{DE} = \left(\frac{1}{n} - 1\right)\rho \ . \tag{8}$$

In the situation when $n = 1$, dark energy component disappears. In other words, observer A wouldn't have perceived any dark energy component if there were *no* underlying structures in universe. Since existence of underlying structures requires parameter values to be $0 < n < 1$ then dark energy component is necessarily *positive*. This is in *contrast* with back-reaction models such as [7, 8] where back-reaction term contributes *negatively* to energy density. As evident, dark energy component is comparable to the magnitude of average energy density ρ. So this model can naturally explain *cosmic coincidence* problem. Another crucial property of dark energy component (8) is that it may appear as *constant* even though it is naively proportional to decreasing average energy density ρ. With the beginning of structure formation, the value of parameter n starts decreasing from unity. So during structure formation proportionality factor $(1/n - 1)$ increases. This implies that for suitable rate of structure formation dark energy component (8) may appear as constant.

For observer A total energy contained within the patch is given by $E = V\rho$ where proper volume of the patch $V = (\pi a^3 L^3/6)$. Using the definition of pressure $P = -(\partial E/\partial V)$, one can derive the *conservation* equation for energy density ρ as

$$\dot{\rho} = -\left(\frac{\dot{V}}{V}\right)\left(\frac{E}{V} - \frac{\partial E}{\partial V}\right) = -3H\rho(1+\omega) \ , \tag{9}$$

where $H := (\dot{a}/a)$ is *Hubble parameter* and $\omega := P/\rho$ is the corresponding equation of state. Analogously, we can define equation of state ω_{DE} for dark energy component such that $\dot{\rho}_{DE} = -3H\rho_{DE}(1+\omega_{DE})$. Using equations (8) and (9), we can compute equation of state for the dark energy

$$\omega_{DE} = -1 + \left[(1+\omega) - \frac{r_n}{3(1-n)}\right] \ , \tag{10}$$

where $(\dot{n}/n) =: -r_n H$. Parameter r_n is a measure of *rate* of structure formation. Dark energy expression (8) along with its equation of state (10) can mimic a cosmological constant at current epoch for suitable values of structure formation parameters n and r_n. In particular, the values of structure formation parameters such that $r_n = 3(1-n)(1+\omega)$ will lead to $\omega_{DE} = -1$ which is the equation of state for a cosmological constant. However, a distinguishing feature between them is that while equation of state for a cosmological constant remains unchanged, the equation of state (10) varies with time.

To explicitly show that structure formation can lead to an accelerating phase, it is convenient to compute Raychaudhuri equation. Taking time-derivative of equation (7) and then using conservation equation (9), we can derive Raychaudhuri equation as

$$3\left(\frac{\ddot{a}}{a}\right) = 4\pi G\left[\frac{r_n}{n} - \frac{(1+3\omega)}{n}\right]\rho \ . \tag{11}$$

From equation (11) it can be seen that the values of structure formation parameter such that $r_n > (1+3\omega)$ will lead to an accelerating phase for observer A.

The modifications to average dynamics of the given patch due to underlying structures, depend on the values of parameters n and r_n. However, the modifications do *not* depend explicitly on coordinate diameters L and l_i. Thus, if one considers a different patch but with *same* values of parameters n and r_n, then one will get *same* Friedmann and Raychaudhuri equations. Given one can pack $\mathbb{R}^3$ space very closely using 3-spheres of *arbitrary* diameters hence the modified Friedman equation (7) and modified Raychaudhuri equation (11) can be considered as good approximation of equations that describe average dynamics of the model universe with underlying structures. Observed contribution from the dark energy component at current epoch is about 70% of the critical energy density. The equation (8) with the parameter value $n = 0.3$ can lead to such an observed amount of dark energy. If we consider the average matter to be pressureless *i.e.* $\omega = 0$ then $\omega_{DE} = -1$ requires parameter value $r_n = 2.1$.

To characterize the matter distribution of inside region as seen by observer B, we need to compute relation between average equation of state ω and equation of state for inside region $\omega_i := P_i/\rho_i$ where pressure $P_i = -(\partial E_i/\partial V_i)$. E_i and V_i are total energy and proper volume of the inside region respectively. As earlier we can derive the conservation equation for inside region, given by

$$\begin{aligned}\dot{\rho}_i &= -\left(\frac{\dot{V}_i}{V_i}\right)\left(\frac{E_i}{V_i} - \frac{\partial E_i}{\partial V_i}\right) \\ &= -3\left(\frac{\dot{a}_i}{a_i} + \frac{\dot{l}_i}{l_i}\right)\rho_i(1+\omega_i) \ . \end{aligned} \tag{12}$$

Coordinate diameter of inside region is time-dependent and it is reflected in conservation equation (12) with its explicit dependence on $(\dot{l}_i/l_i)$. Using the relation between energy density (4), their conservation equations (9) and (12), one can compute relation between the equation of states

$$\omega = (1 - r_n)\omega_i \ . \tag{13}$$

For gravitational collapse to occur with ordinary matter, the corresponding matter distribution must be pressure-less. So for observer B, matter distribution of inside region should be pressure-less *i.e.* $\omega_i = 0$. On the other hand observer A finds average matter also to be pressure-less *i.e.* $\omega = 0$. The equation (13) ensures that physical requirement of observer B and observed fact for observer A can be consistently met. We should mention here that to derive equation (11), one can also use Raychaudhuri equation for observer B. However, in doing so one should be careful to include the additional pressure component coming from the shrinking boundary.

Experimental observations seem to also imply that total energy of the universe has another dark component, the so-called *dark matter* which is pressure-less. From equation (10), one may note that if $\omega = 0$ and $r_n = 0$ *i.e.* if $(-\dot{l}_i/l_i) = (1/n - 1)H$ then the equation of state mimics a pressure-less energy component. It is conceivable that some 'FRW islands' may have different values of structure formation parameters leading to such behavior. However, whether such scenario can explain phenomena ascribed to the presence of dark matter such as galaxy rotation curves, remains to be explored.

To summarize, we have argued that the origin of dark energy can be understood as a consequence of large scale structure formation. This explanation of dark energy does *not* require any exotic matter source nor a fine-tuned cosmological constant. However, presented model in its current form has several deficiencies. Firstly, we assume that structure formation leads to creation of void around each FRW island. However, we know cosmic microwave background (CMB) photons fills up entire universe. Thus, even though CMB contribution to average energy density is negligible during structure formation but for an accurate description one should consider their presence. In this model net effects of structure formation on dynamics of average metric can be summarized by introducing just two characteristic parameters n and r_n. However, the model itself does *not* shed any light on the values of the parameters n and r_n. We may recall that the model is based on sharp-boundary approximation of the matter distribution which is under-going structure formation. Thus, to compute relation between the values of the parameters one needs to perform a detailed simulation of structure formation with a matter distribution which should be then successively approximated by sharp-boundary approximation. Finally, we have not addressed the issue: why is cosmological constant *zero* in our universe?

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